There Is More To Short Barrels Than Just Velocity

Jochem Peelen

14 September 2017

Summary of Findings
This paper presents the effects of short barrels on the ability to hit a given target and on the kinetic energy available to defeat a target. The combination of both is called **firepower** in this context. Comparison is done by simulating shooting at a target of typical size at 400 m for 5.56 mm and 600 m for 7.62 mm. Results are:

- In 5.56 mm NATO, M4 barrel length (368 mm) reduces firepower to **0.71** compared to what the baseline M16 (508 mm) achieves in the same scenario (1.00).
- The 7.62 x 54 R Russian sniper rifle with a 410 mm barrel has its firepower reduced to **0.58** compared to a 600 mm barrel.
- A 7.62 NATO DMR rifle with a 406 mm barrel, compared to a 560 mm baseline, has its firepower reduced to **0.63**.
- Using a 7.62 mm bullet with improved aerodynamics, yet of proven mass-producible design, will **increase** firepower to **1.49**, compared to the common NATO bullet.

1 Introduction

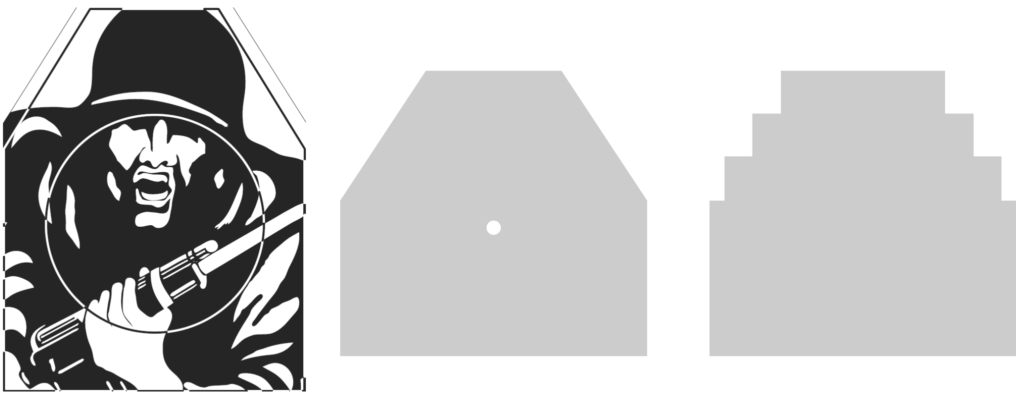

Fig. 1: Hitting a figure 12 target (left, Crown Copyright) at 400 m is the basic scenario discussed. **Middle:** Due to its large size, only upper 400 mm of figure 12 were chosen as the area to be hit. The point of aim (POA) for all shooting (center of mass) is shown. **Right:** Internally used target shape for computation, approximating the sloped lines by rectangles. Presented area is the same.

In 2013 I gave a presentation [21] at Shrivenham which pointed out the dominant role of very large aiming errors in shooting under combat stress. The present paper looks at the other end of the spectrum: a well aimed shot at a difficult target by a soldier who is –at least temporarily– able to keep his stress reactions under control. No doubt, this is also frequently required. It is not always possible to wait for the Designated Marksman.

We all like short, handy and light guns. In close quarters they have obvious advantages. It is a matter of human psychology that short, compact weapons look very cool. Everyone wants one. This way we entered into the age of smokeless propellant with a 800 mm (31.5 in) barrel and have now arrived at 368 mm (14.5 in) barrels being in ever wider use.

The primary effect of shortening the barrel is reducing muzzle velocity. Proponents of short barrels tend to limit comparisons to this single effect. But it has additional consequences:

- A slower bullet has a **more curved trajectory**. As soon as the target distance differs from the sight zero distance, the bullet impact moves up or down, away from the target center. A slower bullet shows more movement of the impact point.
- A bullet, when starting at a slower speed, is **more sensitive to crosswind.**
- The kinetic energy, being the source of target effect (penetration, wounding), drops faster than the velocity alone (by v^2).
- Weapons with very short barrels (for the cartridge used) tend to have greater dispersion than those with normal barrel lengths.

This paper will translate the above observations into concrete numbers.

What is Hit Probability?

To compare the combined effect of trajectory shape and crosswind, we compute the *hit probability* p of two weapons with different barrel length. Hit probability simply is a number between 0 and 1 that tells the user how many of his shots can be expected to hit the target: $p = 0$ means none will hit, $p = 1$ means all shots will hit and a figure like $p = 0.340$ means that 34 out of 100 shots (or 340 out of 1000) will hit the target.

Note that we are dealing with probabilities here, not with predicting the future. Instead of "will hit" we should say "can be expected to hit" the target. The 34 hits mentioned are not guaranteed for any given trial of 100 shots. One may get only 27 hits, but 42 is also possible. Only when a large number of 100 shot trials is done, will **the average** number of hits approach 34.

2 Scenario

U.S. forces have decided to replace the 5.56 mm rifle, simply called M16 here, with a shorter barreled carbine, called M4 here. The M16 has a 508 mm (20 in) barrel. This is the length for which the ammunition was originally designed and is the test barrel length in the NATO standardization agreement. [29] The rifle is 1007 mm long.

The barrel of the M4 carbine is 368 mm (14.5 in) long, 140 mm (5.5 in) shorter. The carbine as a whole is 168 mm shorter. Other nations, like France, also adopted weapons with this barrel length. Therefore, barrel lengths 508 mm and 368 mm will be compared.

3 Ammunition

The ammunition used in this scenario is the 5.56 mm NATO cartridge in its U.S. version M855. For computing the trajectory, muzzle velocity for M16 and M4 is needed as well as the drag coefficient of the bullet.

There is very considerable variation in the published ballistic data. As far as velocity is concerned, measurements done by NATO and published by Per Arvidsson [4] will be used, because they are near the middle of published velocities. For details see page 17. The obtained figures for 508 mm and 368 mm then were reduced to the gun muzzle, using the drag coefficient described in the next paragraph:

- **508 mm** barrel (M16) muzzle velocity (v_0) is 927 m/s
- **368 mm** barrel (M4) muzzle velocity (v_0) is 872 m/s (6 percent less for a 28 percent reduction of barrel length)

Published drag information also varies a lot, partly because different makes of the 5.56 mm NATO bullet show different drag, in some cases 10 percent apart. Most publications still use the so-called *Ballistic Coefficient* (BC or C), which is a computing simplification invented in the 19th century, for use with a large precomputed table of primary functions. BC mixes two independent properties of a bullet, its *sectional density* and its *aerodynamic shape (drag)* into a single number. The BC is not geared to modern computing and nearly all recent programs really use the drag coefficient in their calculations. But as most readers will be more familiar with the BC way of computing, the values that apply here are:

- BC 0.154 (measured by Bryan Litz [14, p. 88] and reported as U.S. Army result [22])
- bullet diameter 0.224 in; mass 62 grains
- this results in a M855 form factor[1] of 1.15

The above numbers are only valid in combination with the G7 aerodynamic drag model, as it is called in the U.S. It actually originated in the UK. Form factor 1.15 means that the drag of the M855 bullet is *on average* 15 percent higher than the G7 model. Modern computer programs ignore BC; they simply use the G7 drag coefficient and multiply it by 1.15. Bullet diameter used in our computation is 5.7 mm and bullet mass 4.0 g. The international standard atmosphere adopted by NATO applies.

4 Target Size and Distance

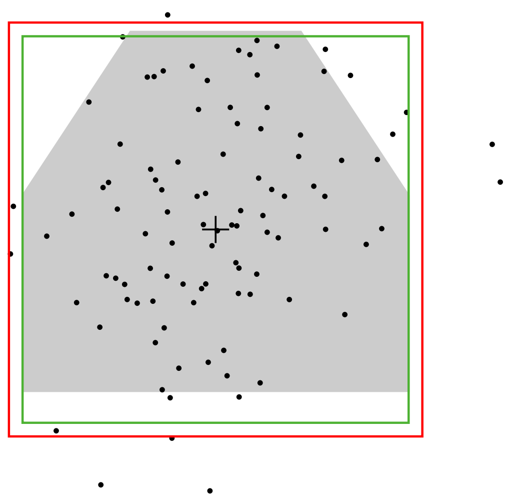

Fig. 2: Optimum situation: target range and sight setting are 400 m. The mean point of impact (MPI, black cross) is at the point of aim. The green frame shows the area covered by 90 % of the impacts from a 508 mm barrel. (Its width being identical to the target is purely coincidental.) Hit probability $p = 0.850$. The red frame shows where 90 percent from the 368 mm barrel weapon are expected. The somewhat larger dispersion results in a hit probability of $p = 0.811$. Black impact points (100, but not all visible) shown here and in the following graphs, were randomly created using the MPI and dispersion of the shorter barrel (red frame).

This work originally started with a UK audience in mind and therefore the UK figure 12 (see page 1) was chosen as target. Doing away with the handsome face on the front side, the entire outline of the target is relevant for hits. A specimen measured[2] was 540 mm high, 430 mm wide at the base and 190 mm wide at the top. The computed center of mass of the target, to be taken as the aim point, is 240 mm above the base (just below the tip of the chin).

Having had a second look at the target, it seemed somewhat large in comparison with other European targets of similar type. These are usually around 400 mm high. So it was decided to use only the top 400 mm of the figure 12 target area. This moves the center of mass (point of aim) 180 mm above the new base.

[1] It is rounded to 2 decimal places because a third decimal place has no perceptible effect in real firings.
[2] Using the middle of the border line, not the paper size.

Because the way hit probability is computed (see appendix 1), the trapezoidal top of the target has to be approximated by some rectangular shape of the same area. In view of the relatively large dispersions expected, three rectangles are considered sufficient.

Distance

Any change in ballistics due to a change in muzzle velocity will not show up at short distances. We need to look at distances closer to maximum effective range. The current U.S. manual [30] does not really give distinct figures in this respect. It only mentions 600 m in passing. [30, p. 3-10 and E-3] Its predecessor [9, p. 2-1] was more clear: 550 m for the M16 and 500 m for the M4. Bundeswehr in the G36 manual [2, p. 7] also lists 500 m. It seems prudent to avoid extremes, so **400 m** is selected as the shooting distance. 400 m is also represented by one of the aiming marks in the M150 Rifle Combat Optic of the M4/M16.

A most important difference between shooting in combat and on the range is that the target distances are not only unknown, but also practically never equal to the range settings of the sight. When the shooter uses the 400 m aiming mark, the target practically always will be closer or farther away.

I am not aware of any data describing the typical mismatch between sight setting and the real target distance in combat situations. To avoid speculation, Ockham's Razor is applied and a very simple model chosen. We start at 350 m and compute hit probability for this distance. This is repeated in 1 m steps up to a distance of 450 m.

The mean of all 101 hit probability values from 350 m to 450 m is taken as indicating the capability to hit the assumed 400 m target.

5 Dispersion and Accuracy

The definitions of dispersion and accuracy, particularly of the latter, may differ from popular views but are as used in ballistics. See the classic by Grubbs [11] or [12, 13, 27].[3]

Dispersion

Group Size, also called *Extreme Spread* (ES), is an easy to apply and therefore generally used measure of bullet dispersion.[4] Alas, it only looks at the two extreme shot holes of a group. The information supplied by the location of the other shot holes is ignored. This makes it very prone to chance. It is unsuitable for what we plan to do.

The measure of error or dispersion used in engineering worldwide is the *standard deviation* (s).[5] This paper will not try to explain what s is or how it is computed. For understanding, it is sufficient to know that the standard deviation is one way to relatively reliably describe the dispersion of bullets.

With the adoption of the 5.56 mm M855 in 1984, the U.S. switched to using the standard deviation [15] in ammunition acceptance for the "accuracy"[6] test. Dispersion can be expressed as a length dimension, for example $s = 66.8$ mm (2.63 in) on a 400 m target. This always requires the inclusion of the target distance. An alternative is expressing it as an angle, which for the same dispersion would be 0.17 mil.[7]

In most cases, using an angle is independent of the distance, because dispersion, like the opening of an angle, grows at the same rate as the distance. While at long ranges the dispersion starts to grow faster, for the range band 350 m to 450 m used here, a constant angle can be assumed.

Ammunition Dispersion

The above example of 0.17 mil is intentional. Specification documents for accepting the M16 [18] and M4 [19] series of weapons prescribe use of M855 ammunition that shows better dispersion than required

[3] Note that the recent publication by Gammon [10] on this subject shows a peculiarity. To compute the standard deviation s of a sample, instead of the usual division by $(n-1)$ the author uses a division by n, as it was popular in the U.S. well into sixties of the 20th century. But nowadays, division by $(n-1)$, as described in [12, p. A-4] for example, is general standard.

[4] Precision is another term for dispersion.

[5] Readers who do not like this name should blame Karl Pearson. He proposed it in 1894.

[6] The specification calls it accuracy, because that is the name that has been used all the time in acceptance requirements. What is really measured is the *dispersion*, by shooting 3 groups of 30 shots each.

[7] Mil is an internationally used military unit of angular measurement, corresponding to 3.375 MOA (true MOA, not Shooter's MOA). Note the existence of a subtle difference between mil and mrad.

by the M855 acceptance specification. Translating the weapon acceptance figures to angular standard deviation yields 0.17 mil.[8] **0.17 mil ammunition dispersion** (horizontal and vertical) is assumed.

Rifle Dispersion

To fire at a target like the one described above, the prone supported position will be used and it is assumed that the weapon has an optical sight with a magnification of about 4x. A European trial under similar circumstances resulted in a weapon dispersion of s_t = **0.28 mil** (horizontal and vertical) on the target. [22]

Carbine Dispersion

As a general rule, variations in barrel length do not change bullet dispersion in a measurable way. Unless one goes to extremes. Long, thin barrels may suffer from barrel whip, which may increase dispersion. In that case a shorter barrel can even be advantageous, because it is relatively stiffer.

But if it gets too short, irregularities in propellant burning from shot to shot may affect dispersion. Significantly higher pressures at the gas port, causing a more violent bolt operation, and at the muzzle may also come into play. In any case, a 368 mm barrel already seems to produce negative effects, as the specifications for M16 and M4 show. For the M16 series of rifles, the acceptance criterion (called "accuracy and targeting") is a maximum extreme spread of 5 in for 10 shots at 100 yd. [18] Originally, this was also the M4 carbine requirement. But to reflect the real world M4 behaviour, it had to be increased to 5.6 in extreme spread. [19]

The acceptance firing is done from a machine rest. The obvious conclusion is that the M4 carbine, although firing the same ammunition, has a dispersion about 12 percent larger than the M16 rifle.

How can this be translated into a meaningful standard deviation? Simply increasing the 0.28 mil of the rifle by 12 percent would not be correct. The 0.28 mil value is a trial result, containing ammunition and shooter error. Ammunition error s_a is 0.17 mil. We will assume shooter error, when taking a well aimed shot from a stable position, using a magnifying optical sight as $s_s = 0.10$ mil (compare [25, p. 100] and its translation [26, p. 119]).

The details of how to compute the carbine dispersion are explained in an appendix on page 16. The result is a dispersion of **0.30 mil**.

Accuracy

Accuracy describes the distance of the shot group center (*mean point of impact*, MPI) from the *desired point of impact* (DPI), which in this case is the *point of aim* (POA). Accuracy is a basic precondition to be able to hit a target. Unless the weapon shoots where you aim, even the smallest dispersion is useless. Accuracy means the weapon must be zeroed. In this scenario, a weapon **perfectly zeroed** at 400 m is assumed.

It has to be admitted that this assumption is definitely not true in real life. Zeroing in itself is very problematic, because of

- the difficulty to determine the true zero of a weapon. Bullet dispersion, which is a random process, blurs the true location of the MPI.
- Every soldier has an indiviual view of the sight. The same, unchanged weapon, when used by different experienced shooters, will show a different MPI for each of them. This is more pronounced with mechanical than with optical sights.
- A great majority of soldiers is unable to shoot consistently enough to obtain a good zero on their weapon.

In view of these problems, the change of MPI due to different ammunition lots, although mentioned very often, is in my experience a non-issue. Our dilemma is between the inability of the average shooter to zero his own weapon to a satisfactory degree and the impossibility to let others –or a machine– do the zeroing, because the MPI always is different. The result are weapons with zeroes that fail to achieve the desired accuracy.

[8] For M855 ammunition acceptance, the maximum allowed standard deviation originally was the equivalent of 0.37 mil [15], later tightened to 0.32 mil. [16] The NATO STANAG is less strict at 0.38 mil. [29] Bundeswehr requires the equivalent of 0.22 mil [22], although measured at 100 m, not 548 m as is done at Lake City.

The unsolved question is, what quality of zero is practically achieved? The problems of zeroing have, to my knowledge, never been researched by people with at least some basic experience in rifle shooting at medium ranges. Otherwise, armies would not have adopted "zeroing" at 25 m in the belief that this produces a useful result in a more economical way. At least the U.S. Army, obviously after learning the hard way, in its new manual [30, p. E-1] clearly states that a zero has to be confirmed "out to 300 m."

In short, we have no data about the quality of zero to be found in the hands of troops. Therefore, again, Ockham's Razor is applied and a "perfect" zero for 400 m with the ammunition at hand is assumed. This is the assumption with the least amount of speculation in it.

In computing the MPI position, the height of the line of sight above the muzzle must be taken into account, although its effect is usually overestimated. Here it is taken as 66 mm.

6 Effect of Trajectory Curvature

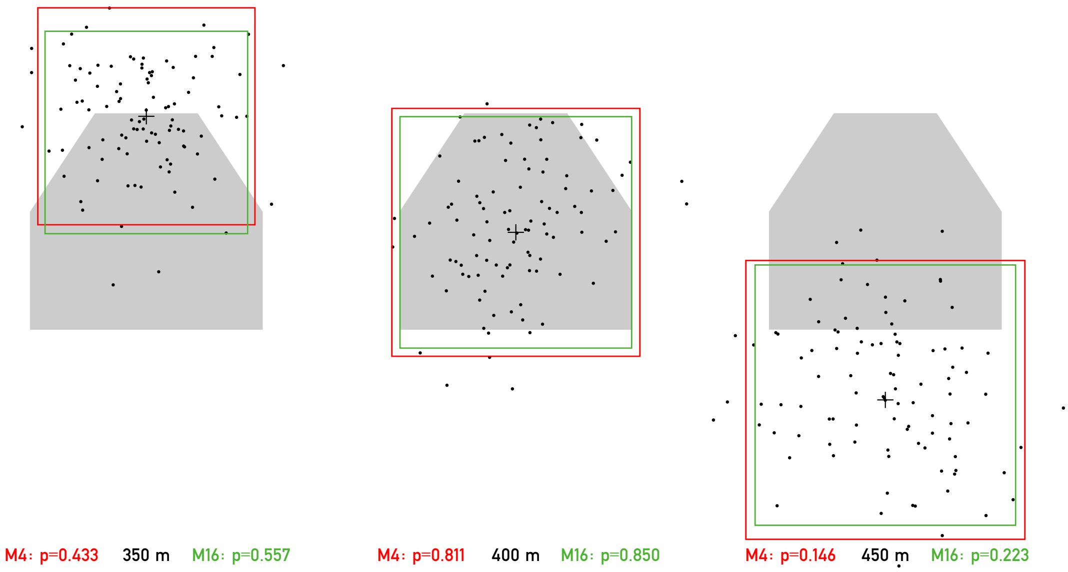

Fig. 3: Effect of trajectory curvature. When the target is nearer as expected (left, 350 m), impacts are high. Only at the correct range (middle, same as figure 2) the MPI is at the POA as desired. When the target is farther away (right, 450 m), the shots go low. Because drop velocity is faster, the shift of impact is larger. MPI shift is generally larger for the shorter barrel (vertical offset of red frames), because the trajectory is more curved. p is the hit probability computed for each distance.

Gravity pulls on the bullet all the time. A zeroed sight means we are shooting upwards by the exact amount necessary to compensate for bullet drop during its flight to the target. The sight setting is only correct for one target distance, 400 m in our scenario.[9]

If the target is nearer than the sight setting, the bullet has not yet dropped by the expected amount. Impact is above the point of aim. Should the target be farther away, the bullet spends more time dropping. Impact is below the line of sight.

A bullet from a shorter barrel is slower, has a longer time of flight between two given points and consequently more drop. The described change in impact location is therefore greater. Figure 3 shows this effect for our scenario (red frames).

[9] Because the muzzle is 66 mm below the line of sight, the bullet crosses the line of sight twice. First on the ascending branch of the trajectory at a distance of 17.4 m for the M4 and 19.9 m for the M16. The second crossing happens on the descending branch at 400 m in our scenario. Strictly speaking, the sight setting is correct for two distances.

7 Effect of Crosswind

On a given day, crosswind mostly comes from the same general direction and changes speed in an unpredictable fashion, sometimes from shot to shot. How far a bullet is driven sideways by crosswind again mostly depends on the time of flight. Calculations show that the same bullet, when flying slower due to a shorter barrel, is more affected by crosswind than at higher speed.

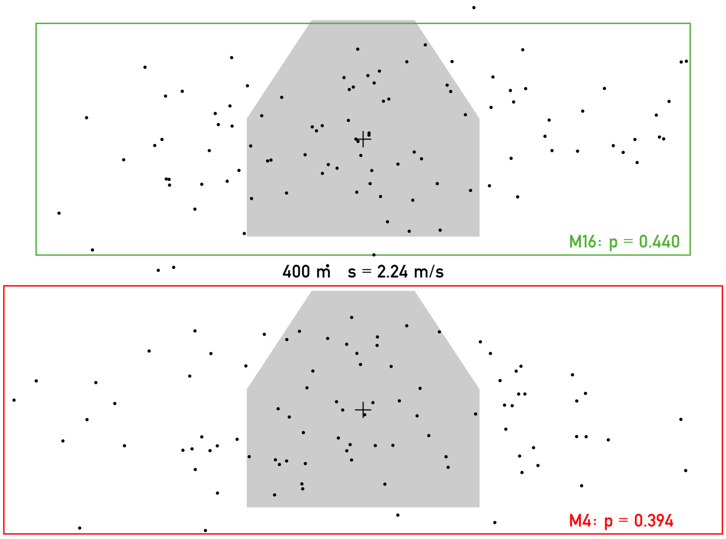

Fig. 4: Comparing the effect of 2.24 m/s (7.35 fps) standard crosswind on the resulting horizontal dispersion, as used for the hit probability computations. The frames show the area of 90 percent of all shots.

What we need is a way to express crosswind sensitivity of a bullet that shows its effect on hit probability. Luckily, the other big direct fire community, that of battle tanks, has found a simple method for this.

- The common experience of crosswind coming from about the same direction over some time is dropped.
- Assuming that the wind may come from both sides with equal probability, we can look at it the same way as one of the other random disturbances, like aiming error for example.
- As in the example of aiming error, mean crosswind is taken as zero. The intensity of crosswind is expressed by a standard deviation, for example 2.24 m/s (7.35 fps).
- The wind variation distribution is in effect simulated by a normal distribution.
- This wind standard deviation is conveniently called *standard crosswind*.

This allows to enter crosswind simply as an **additional horizontal dispersion** into our scenario. How this is done is explained on page 16. For example at 400 m, horizontal standard deviation with crosswind is **0.3106 m**, while it would be 0.1100 m without it.

The 2.24 m/s figure chosen here is an average from U.S., British, Soviet and East German sources. [20] It is smaller than the well known U.S. figure of 3.35 m/s (11 fps), because the latter is based on random 360 degree wind directions, not limited to the crosswind component.

8 Cant

Gravity always makes the bullet drop vertically. The purpose of the sight is compensating bullet drop, consequently it must be aligned vertically, too. A weapon is called canted when this is not achieved. U.S. experiments in a simulator environment [24] showed an average cant of 3 degrees to the left.

Cant moves the mean point of impact sideways and downwards. The amount depends on the angle of departure. With small arms, the downward movement is usually not larger than a bullet diameter. In our scenario, the bullet drop at 400 m, due to the angle of departure, is 1308 mm (long barrel) and 1496 mm. Then a 3 degree cant causes 68 mm and 78 mm sideways movement, but only 1.8 mm and 2.1 mm downwards movement of the MPI.

Considering 430 mm target width, this sideways movement doubtless should not be ignored. But the program used for computing hit probability at this time is not able to cater for cant. In theory, it would have been possible to simulate cant effect by manually changing the point of aim for each of the 101 distances per run. But that would have been prohibitive. Therefore, the results presented here ignore cant effects.

9 Kinetic Energy: Target Effect

To be clear: the **location** of a body hit by far outweighs all other considerations, particulary in situations where rapid incapacitation is desired. Hitting an unimportant part of the body simply means absence of rapid incapacitation, whatever the bullet design or energy.

But generally, for small arms bullets of similar caliber and design, the kinetic energy is the dominating source of wounding power as well as effects against material targets. Kinetic energy changes with the velocity squared. From a 508 mm barrel the M855 bullet (927 m/s) has a kinetic energy of 1719 Joule.

The 6 % lower muzzle velocity from a 368 mm barrel (872 m/s) delivers 1521 Joule, which is 12 % less. This is a general rule: expressed in percent, a change in velocity causes a change in energy that is two times larger.

Expressed in terms of range, the difference in kinetic energy is equivalent to 52 m. Assuming that a given target, due to bullet energy, can be penetrated at 100 m from a 508 m barrel, the same effect from a 368 mm barrel ends at about 48 m. This relation applies to longer ranges, too (400 m reduced to 348 m).

10 Firepower Definition

To defeat a target, it must not only be hit, but enough kinetic energy delivered to it.[10] Less kinetic energy, compared to the baseline system, means less target effect. This is a simplification, but true when comparing the same bullet at similar velocities.

For the purpose of this paper, we define **firepower** as a combination of the index figures for hit probability and kinetic energy. It is computed by multiplying both.

11 Main Result: 5.56 NATO

Figure 5 on page 9 at last shows the result of the computations done. The *average* hit probability for the 350 m to 450 m range band, computed in 1 m steps, is:

- for the 508 mm barrel weapon: $p = \mathbf{0.347}$
- for the 368 mm barrel weapon: $p = \mathbf{0.291}$

Using the existing 508 mm weapon as a baseline (index figure 1.0), the index figure for the shorter barrel weapon muzzle velocity is 0.94, reducing kinetic energy at 400 m from 603 Joule to 515 Joule. Doing a baseline comparison, the index figures for the short barrel are:

- hit probability index: **0.84**
- 400 m kinetic energy index: **0.85**
- firepower index (0.84 $\times$ 0.85) drops to: **0.71**

We find, the shorter barrel leads to a loss of nearly 30 % of firepower at 400 m. This loss of ability to hit the target and, if hit, of target effect, is too large to be ignored.

Effect of Variations

When working on this paper, I tested the effect of a number of variations of the parameters. For example, a somewhat smaller bullet dispersion (for shooting from a machine rest instead of prone supported). I

[10] DOD-NATO definition of firepower: "amount of fire; ability to deliver fire" according to [6], but undefined in [7].

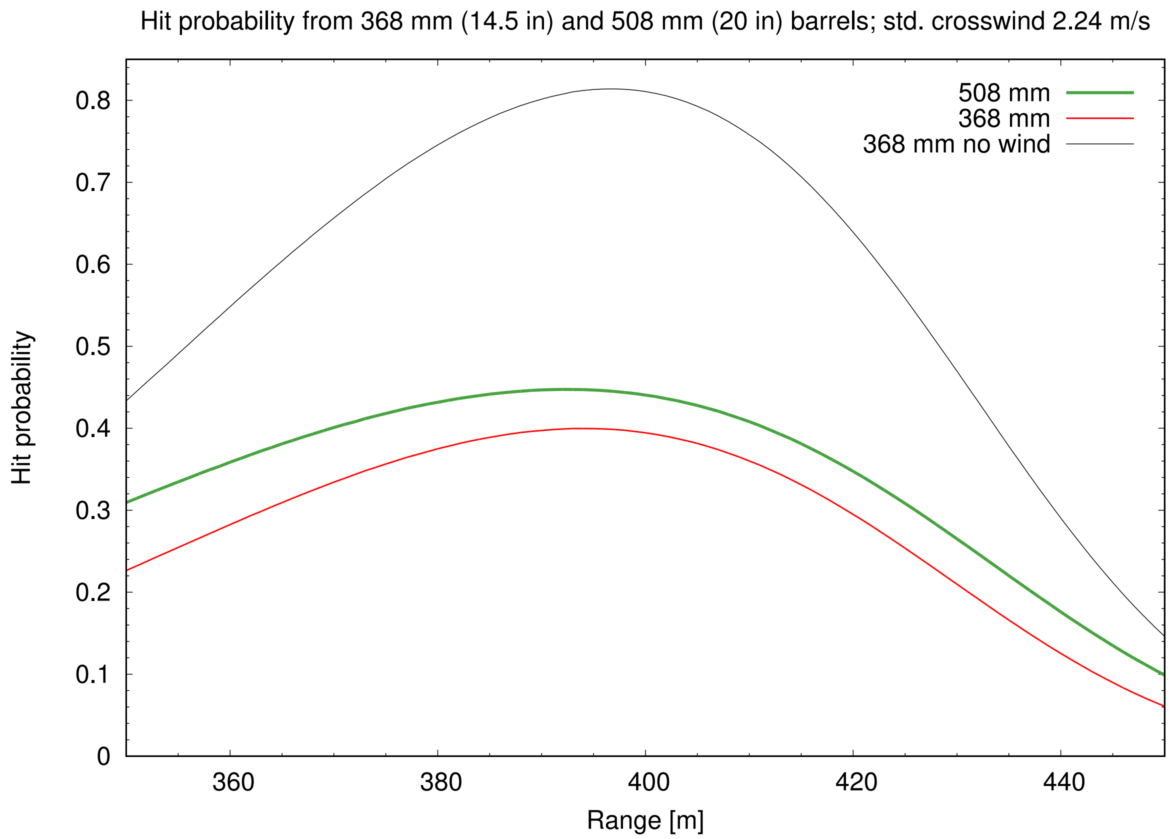

Fig. 5: The **main result** of this paper. M4 (368 mm barrel) hit probability (red line) clearly is inferior to the M16 (508 mm barrel; green line) in our scenario. The distance to the black line illustrates how big the effect of crosswind at this range is.

also checked a very differently shaped target used in Sweden, first mentioned by Per Arvidsson [3]. Its a narrow rectangle 120 mm wide and 390 mm high, not at all like the modified figure 12 used here. Other checks were increasing and decreasing the muzzle velocities by 15 m/s.

It turned out that with all these modifications, the relation between long and short barrel did not change very much. The hit probability index figure for the short barrel remained in the 0.84 to 0.87 range.

12 Russian Short Barrel 7.62 mm Sniper Rifle

Russian small arms industry recently announced the new Tschukawin (Chukavin) sniper rifle. Possibly inspired by the very short 420 mm (16.5 in) barrel of the German 7.62 mm G28 rifle, the Russians offer their sniper rifle with a 410 mm (16.1 in) barrel. This is considerably shorter than the Mosin carbine of 1944, which had a 517 mm (20.4 in) barrel. This Mosin is already quite hard on the shooter.

The excellent precision of the G28 is known. But as was shown above, trajectory shape and crosswind sensitivity are independent of bullet dispersion. The question is, how barrel length affects the effectiveness of sniper rifles.

Because we are dealing with a sniper rifle, the range considered in the scenario is now **600 m**. Target and wind conditions remain the same. An important change is choosing a much smaller bullet dispersion of **0.10 mil** (horizontally and vertically), considering excellent shots at work. Figures in Russian publications [8, vol. 4, p. 430] describe the 7N14 sniper cartridge as equivalent to 0.06 - 0.09 mil standard deviation.

Estimating the effect of barrel length on bullet velocity is based on measurements done by Clark, [5] (see page 18 for details), which, when compensated for the first 3 m, result in:

- **600 mm** barrel muzzle velocity 830 m/s
- **410 mm** barrel muzzle velocity 763 m/s

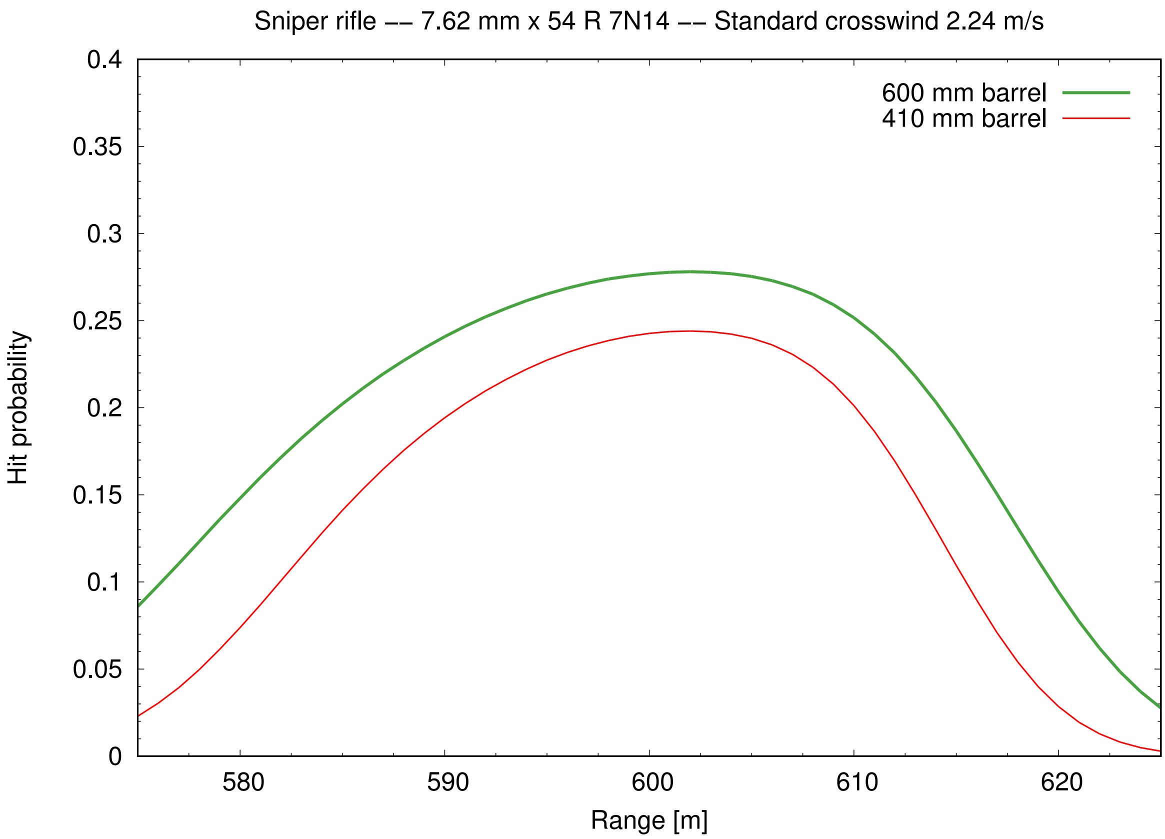

Fig. 6: Effect of barrel length on a sniper rifle at 600 m. The loss of hit probability for the 410 mm barrel (red line) compared to the baseline 600 mm barrel (green line) is only shown between 575 m and 625 m, because beyond that, hit probability is practically zero. Note the changed vertical scale from figure 5.

Clark used Bulgarian surplus ammunition with the standard 9.6 g bullet mass. From a Russian source [31, p. 178] we know the 7N14 sniper round has a 9.95 g (153.5 gr) bullet with about the same muzzle velocity. For lack of other information, the above velocities are used here.

No detailed velocity/distance data of the 7N14 is known, let alone drag measurements. The shape [31, p. 96] looks very similar to the ordinary LPS. For this reason a G7 form factor of 1.08 is assumed. The computation is done with a bullet diameter of 7.9 mm, the already mentioned mass of 9.95 g and a muzzle position 78 mm below the line of sight.

7.62 x 54 R Result (7N14)

Figure 6 shows the outcome of the computations. At 600 m, the range band observed was reduced to ±25 m, because hit probability practically drops to zero beyond that band. The *average* hit probability between 575 m and 625 m is:

- for the 600 mm barrel weapon: $p = \mathbf{0.198}$
- for the 410 mm barrel weapon: $p = \mathbf{0.145}$

At the same time, the kinetic energy at 600 m drops from 1005 Joule to 798 Joule. Doing a baseline comparison, the index figures for the short barrel are:

- hit probability index: **0.73**
- 600 m kinetic energy index: **0.79**
- firepower (0.73×0.79) drops to: **0.58**

The shorter barrel leads to a loss of more than 40 % of firepower at 600 m.

13 7.62 NATO Short Barrel

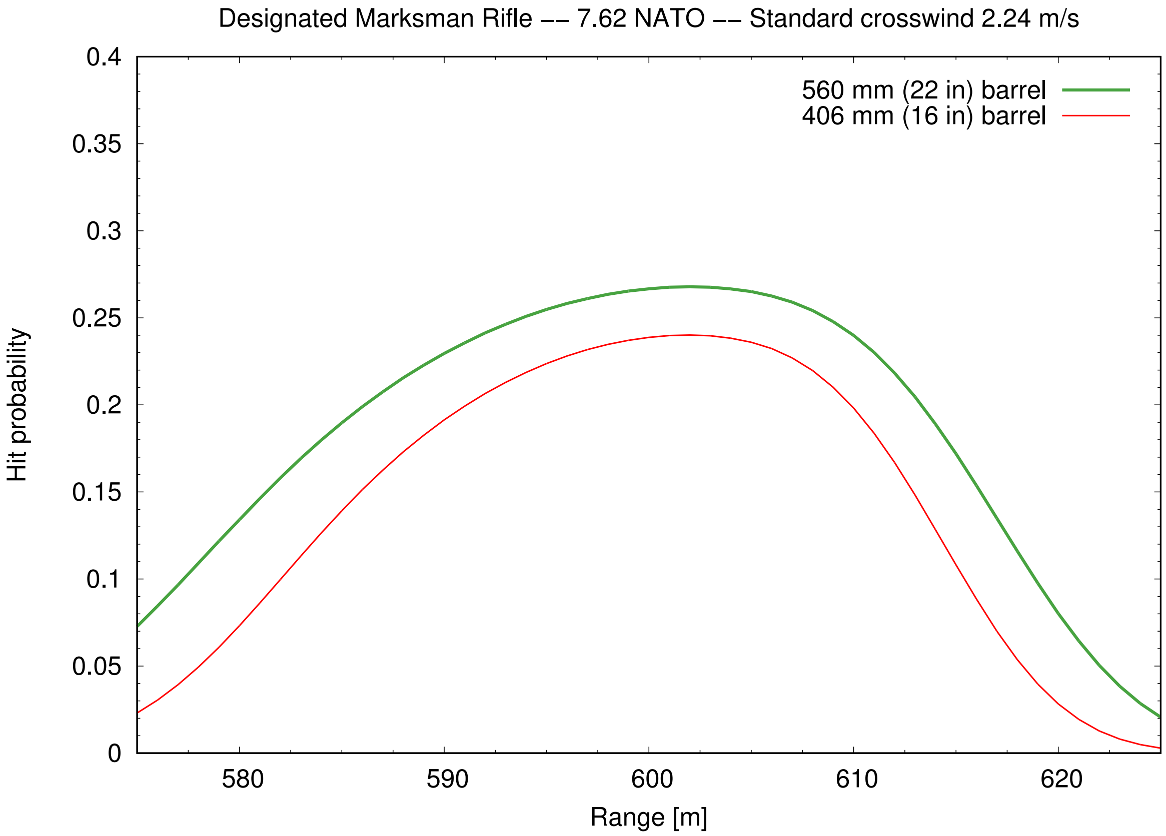

Fig. 7: Effect of barrel length on a 7.62 NATO DMR at 600 m. Due to the 7.62 NATO, origin in 1954, and the Russian 7.62 mm rifle cartridge, origin in 1908, being nearly equally powerful, the resulting hit probability curves are quite similar to figure 6. The 406 mm barrel (red line) clearly reduces hit probability compared to a 560 mm barrel (green line).

As written in the previous section, the Russians seem to have taken their inspiration for a sniper rifle with a very short barrel from weapons offered in the West, like the G28 (420 mm, 16.5 in) Designated Marksman Rifle (DMR). Short barrels on precision rifles are a rather new development. Heckler & Koch actually had a 650 mm (25.6 in) barrel version of the G28 in development [23], the same length as found on the PSG1 police sniper rifle. The G28 predecessor MSG90A2 had a 600 mm (23.6 in) barrel.

But today, 406 mm (16 in) barrels for 7.62 NATO DMRs are seriously considered. In this section, the effect of this barrel length will be compared to the standard barrel length of 7.62 NATO, which is 560 mm (22 in). [28]

To estimate the effect of barrel length on bullet velocity, measurements published in `rifleshooter.com` were used. See page 19 for details. We get:

- **560 mm** barrel muzzle velocity 828 m/s
- **406 mm** barrel muzzle velocity 772 m/s

The STANAG on the 7.62 mm NATO [28] allows considerable latitude of bullet characteristics (mass between 8.4 g and 10.0 g for example; 130 to 154 gr). But only quite similar designs are in use. Drag data is plenty, but varies between G7 form factors 1.02 and 1.15. Here an average form factor of 1.08 is assumed, the same as used for the Russian 7N14. The computation takes bullet diameter as 7.8 mm and mass as 9.45 g (146 gr).

The *average* hit probability between 575 m and 625 m is:

- for the 560 mm barrel weapon: $p = \mathbf{0.186}$
- for the 406 mm barrel weapon: $p = \mathbf{0.143}$

At the same time, the kinetic energy delivered to a target at 600 m drops from 912 Joule to 752 Joule. Doing a baseline comparison, the index figures for the short barrel are:

- hit probability index: **0.77**
- 600 m kinetic energy index: **0.82**
- firepower (0.77 × 0.82) drops to: **0.63**

The shorter barrel leads to a loss of nearly 40 % of firepower at 600 m. This loss is a little less than the result for the Russian cartridge. It is caused by the somewhat higher muzzle velocity of the 7.62 NATO from the short barrel compared to the 7.62 mm Russian cartridge, while for the longer barrel the velocities are about equal.

14 Trying an Alternative Approach: Better Aerodynamics

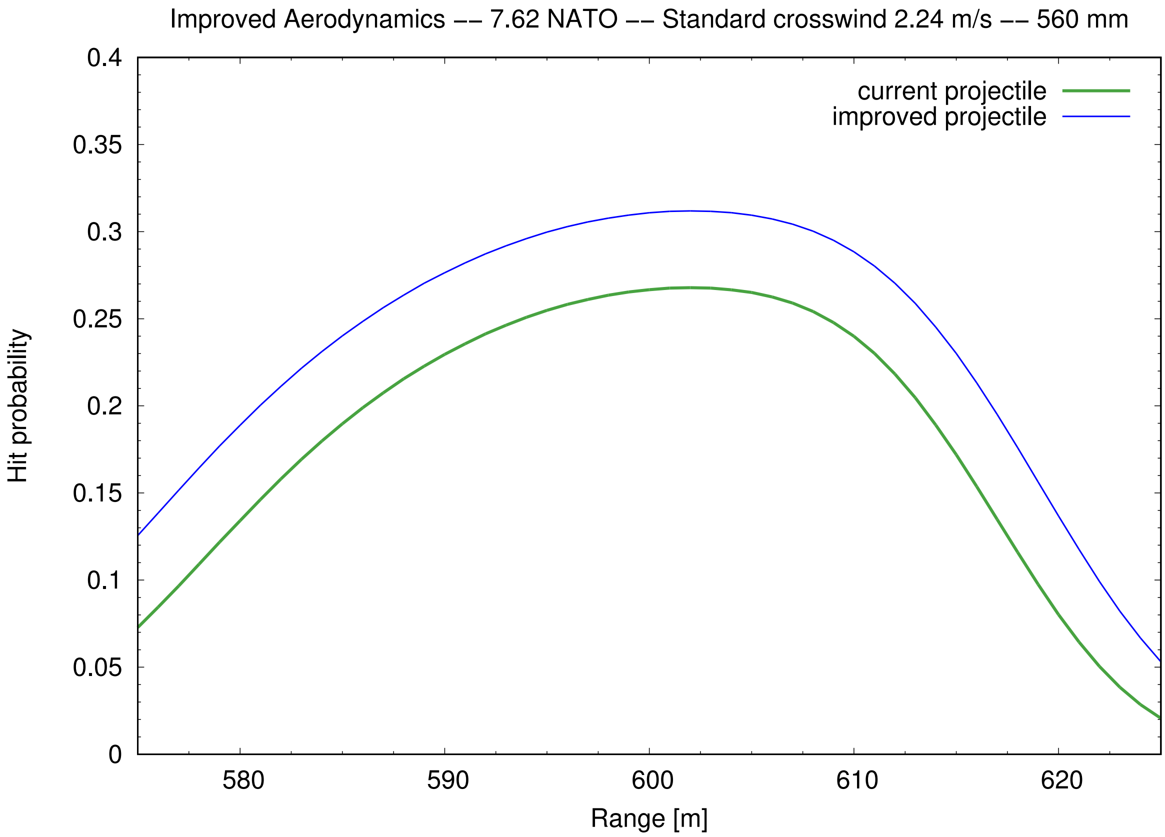

Fig. 8: Effect of improved aerodynamics on hit probability of a 7.62 NATO DMR with a 560 mm barrel at 600 m. The improved aerodynamics (blue line) is that of the French 33D machine gun bullet, for which modern radar drag data is available. The green line is the same as in the previous figure.

Dealing with the exterior ballistics of current military small arms bullets leads to the conclusion that their drag figures are, well, average. There is of course the requirement to be mass-producible without too much difficulty. A bullet shape that is easy to make, usually is at odds with what the aerodynamicist would like to see.

But an aerodynamically better bullet will have less drag, resulting in a flatter trajectory and have reduced sensitivity to crosswind. Kinetic energy delivered to the target will be higher. Could aerodynamically improved bullets add to our firepower?

To answer this question, the 7.62 mm scenario with the 560 mm barrel was recomputed with an improved bullet. To isolate the aerodynamic effect, bullet **mass and muzzle velocity are the same** as for the ordinary NATO bullet.

The aerodynamic gain is not from some exotic design like Voss or Haack, which pop up again and again in discussions, but never make it to practical use. I took a design proven to be mass-producible and lacking any fancy features: the French 33D machine gun bullet. It has the same diameter of 7.8 mm. This guarantees the absence of scaling effects on the drag. We have reliable drag data for it, because modern radar measurements by ETBS Bourges [20] are available. This radar data is comparable to a G7 form factor of 0.94 on average.

The blue curve in figure 8 shows the result. The *average* hit probability between 575 m and 625 m with a 560 mm barrel for both bullets is:

- for the ordinary NATO bullet: $p = \mathbf{0.186}$
- for the improved bullet shape: $p = \mathbf{0.235}$

The kinetic energy delivered to a target at 600 m raises from 912 Joule to 1079 Joule. Doing a baseline comparison again, the index figures for using a bullet with improved aerodynamics are:

- hit probability index: **1.26**
- 600 m kinetic energy index: **1.18**
- firepower (1.26 $\times$ 1.18) is increased to: **1.49**

> Firepower would go up by close to 50 % just by improving the current 7.62 NATO bullet. Frankly, this amount of improvement surprised me.

Having said that, it has to be kept in mind that aerodynamic improvements for small calibers like 5.56 mm are much harder to achieve than for larger calibers. Look at the match bullet data compiled by Litz [14], for example. It is obvious that for a given bullet length[11] the larger calibers have smaller drag coefficients than 5.56 mm. This is a clear indication of the difficulty to make small diameter bullets as aerodynamically efficient as medium and large calibers.

15 Closing Remark: Economizing on Reticles

Some Bundeswehr units use a short barrel version of the G36, called G36K. I noted that, to my surprise, the same standard optic is being used on both. The U.S. have a similar approach. I found no hint that there might be a version of the M150 Rifle Combat Optic for the M16 and different one for the M4.

Doubtless, this saves money and logistic burden. On the other hand, as we have seen, bullet trajectories are not identical. What is the effect on the ability to hit a target?

These optics have no range setting knob, they use a reticle with distinct aiming marks for several distances, the shortest often being 100 m. This old invention is nowadays called *Bullet Drop Compensator* (BDC). Let us assume a reticle perfectly zeroed at 100 m. That means, we use the 100 m line of sight as a reference. The reticle has a 400 m aiming mark that must match bullet drop at that range.

Computation shows that for the M16, at 400 m the MPI is 862 mm (33.9 in) below the 100 m line of sight. We can assume, the 400 m aiming mark in the reticle is located accordingly.

What happens, when the same reticle layout is used on a M4 carbine? After correct zeroing, a dead center MPI at 100 m will result. But, the short barrel bullet drop at 400 m is larger, being 1018 mm (40.1 in) below the 100 m line of sight. When using a 400 m aiming mark calibrated for the M16 trajectory, the M4 shoots 156 mm (6.14 in) low. When shooting in wind, the effect on hit probability at 400 m is:

- M4 with reticle calibrated for M4 368 mm barrel: $p = \mathbf{0.440}$
- M4 with reticle calibrated for M16 508 mm barrel (shooting 156 mm low): $p = \mathbf{0.262}$

Capability to hit a target is significantly reduced by using the same "bullet drop compensator" reticle for different barrel lengths.

[11] Expressed in bullet diameters, sometimes called fineness ratio.

16 Appendix 1: Principle of Hit Probability Computation

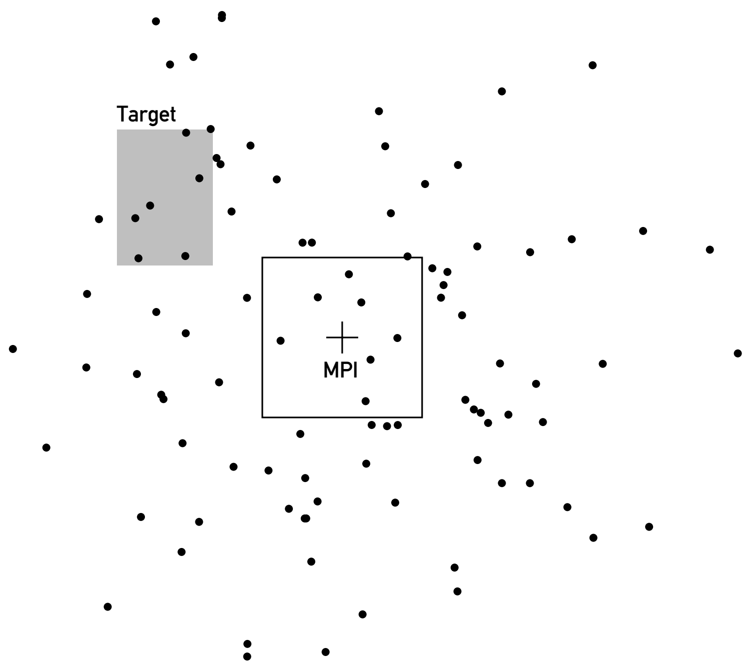

Fig. 9: The mean point of impact (MPI) is the starting point for computing hit probability. All dimensions are to be converted in relation to **1 standard deviation**, which is the edge length of the square. The target is 0.6*s* wide and 0.85*s* high.

Without using mathematical formulas, this appendix will try to explain **the principle** that makes it possible to compute the number of hits we can expect on a target in a specific scenario. Necessary conditions are:

- The bullet dispersion (vertically and horizontally) must follow the so-called *normal* or Gaussian[12] distribution. While this is definitely not the case for shooting under stress and time pressure, it is generally accepted as true for slow aimed shooting by an experienced shooter from a supported weapon. Its also true for shooting from test barrels.
- Bullet dispersion must be expressed as *standard deviation.*
- Accuracy: the mean point of impact (MPI) must be known.
- There is no correlation between hit locations. Horizontal and vertical dispersion must be independent of each other.

To understand the process, we must turn our usual view of the target as being at the center of events and the shot holes on its periphery, upside down.

> We make the MPI the central point of our observations and "1 standard deviation" the basic unit of all measurements.

Whatever the real dimension of the target, be it a silhouette, a barn door or a tank, its dimensions must be converted into standard deviation units. If the standard deviation of the dispersion is 176 mm and the target size 250 mm, then target size is 1.42 for the computation. Horizontal and vertical distances from target to MPI have also to be converted the same way.

As figure 10 indicates, we know the horizontal distance of lines **L** and **R** from the MPI (left and right target edge). These numbers make it possible to compute how many shots can be **expected to hit the**

[12] As so often in mathematics, it is attributed to the wrong person. This distribution was first described by *Abraham de Moivre* (1667-1754) who had escaped to London from religious persecution in France.

stripe delimited[13] by **L** and **R**. The mathematical tool for this is called *Cumulative Distribution Function* (CDF). It can be computed by software with astonishing precision. Let us assume a CDF result of 0.20, meaning that 20 out of 100 shots on average will hit[14] the vertical stripe between **L** and **R**.

The same computation is then repeated for the horizontal stripe **T** and **B**, derived from the top and bottom target edges. Let us assume a CDF result of 0.25. On average, 25 out of 100 shots will hit the horizontal stripe.

Now, to hit the target, a shot must *at the same time* horizontally fall within **L** and **R** and vertically between **T** and **B**. Because both dispersions are independent of each other, the probability of such an event can be computed surprisingly easy by multiplying 0.20 by 0.25, resulting in 0.05.

The target shown in figure 9 has a **hit probability of 0.05**. What we see in figure 9 among 100 simulated shots of the correct dispersion, is 6 hits (the impact at the top right is outside). This is an example that hit probability is **not** an **exact prediction of a single event**. Other 100 shot groups will place more or fewer hits within the target. Over a large number of repetitions, the average number of hits will approach 5.

Because we look at horizontal and vertical dimensions only, sloped target edges cannot directly be handled this way. Their position and size has to be approximated by sufficiently small rectangles and squares.

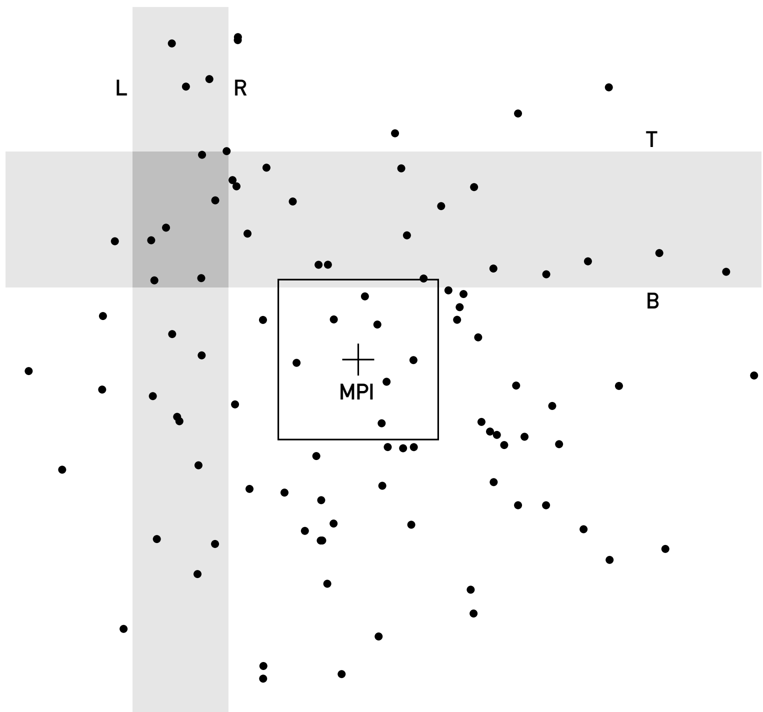

Fig. 10: When the horizontal distance of the lines **L** and **R** from the MPI is known, it is possible to compute the proportion of hits within stripe **L R**. The same is possible for the vertical distances of lines **T** and **B** from the MPI. The product of both numbers is the hit probability for the target rectangle.

Another approach, on the surface more elegant, is to describe dispersion by circular coordinates, which have expanded to an elliptical model when horizontal and vertical dispersions are not same. It is extremely easy indeed to compute the hit probability for a circular target in a circular dispersion *as long as both centers are the same.* But computing any other arrangement is by no means trivial and becomes much more challenging if ellipses are involved. Therefore, the rectangular approach is used here.

[13] Vertically, the stripe is mathematically bounded by infinity only. But we can ignore that.
[14] Hit or not is determined by the location of the bullet center, not its diameter.

17 Appendix 2: Adding and Subtracting Dispersions

Using standard deviations instead of group sizes makes it possible to isolate effects of single components of a given dispersion. This is done by applying the *rule of error propagation*.

Computing Carbine Dispersion

On page 5 we encountered the problem of translating the 12 % larger acceptance dispersion of the M4 into a meaningful dispersion on the range. Simply increasing the 0.28 mil of the rifle by 12 percent cannot be done. The 0.28 mil value is a trial result, containing ammunition and shooter error. Ammunition error s_a is assumed as 0.17 mil. Shooter error, when taking a well aimed shot from a stable position, using a magnifying optical sight is taken as $s_s = 0.10$ mil (compare [25, p. 100] and its translation [26, p. 119]).

Stripping these two numbers from the rifle trial dispersion s_t is done by applying the rule of error propagation:

$$s_r = \sqrt{s_t^2 - s_a^2 - s_s^2} = \sqrt{0.28^2 - 0.17^2 - 0.10^2} = 0.199$$

The isolated *rifle* error s_r is 0.199 mil. We increase it by 12 percent to get its carbine equivalent. This is admissible, because computer simulations have shown that the relationship between ES and s is linear. If the ES of a large number of 10 shot groups is 12 percent greater, the standard deviation of these shot groups is by the same amount greater. Conversion of the rifle error s_r into the carbine error s_c is therefore simply done by:

$$s_c = 1.12 \times s_r = 1.12 \times 0.199 = 0.223$$

Merging it with ammunition and shooter error, again via the rule of error propagation, tells us the estimated standard deviation s_E of the carbine dispersion:

$$s_E = \sqrt{s_c^2 + s_a^2 + s_s^2} = \sqrt{0.223^2 + 0.17^2 + 0.10^2} = 0.298 \approx 0.30$$

Dispersion Due to Crosswind

The rule of error propagation also makes it possible to compute horizontal dispersion due to crosswind. At each distance, drift due to 1 m/s crosswind is computed the conventional way by the trajectory program. Using the 508 mm barrel, at 400 m the drift due to 1 m/s crosswind is 0.1297 m. Assuming 2.24 m/s standard deviation of the crosswind, this results in

$$s_w = 0.1297 \times 2.24 = 0.2905$$

where s_w is the standard deviation of horizontal dispersion due to crosswind. This dispersion is joined with the "no wind" horizontal dispersion for 400 m (0.1100 m in this example):

$$s = \sqrt{s_x^2 + s_w^2} = \sqrt{0.1100^2 + 0.2905^2} = 0.3106$$

The 400 m horizontal dispersion with crosswind is 0.3106 m standard deviation.

18 Appendix 3: Estimating Velocity from Barrel Length

It is often desirable to estimate bullet velocity from a given barrel length. This appendix describes how it was done for this paper. The velocities can be taken with a ruler from the smooth curves in the diagrams or computed via the simple equations given. The vertical scatter of the points representing each shot in the 7.62 mm diagrams should be a reminder of the considerable shot-to-shot variation in real shooting. I am aware that the resulting velocities are lower than typically listed in promotional material. But to my knowledge they are realistic.

5.56 NATO Bullet

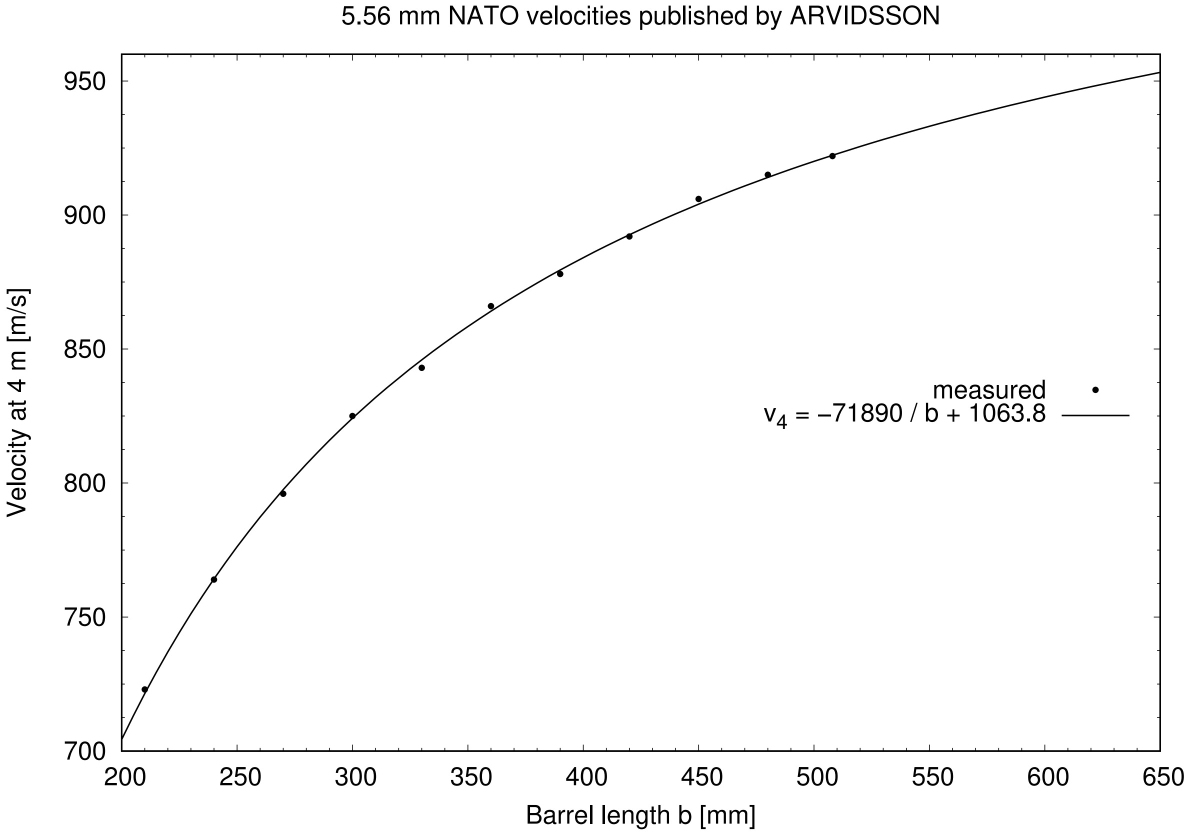

Fig. 11: Tool for estimating 5.56 mm NATO velocity v_4 from barrel length. The source data was published by Arvidsson [4]. Be careful when using the extrapolation beyond 508 mm to 650 mm barrel length.

Per Arvidsson published [4] measurements done by NATO. His table lists velocities measured 4 meters in front of the muzzle, down to 210 mm barrel length. In view of the great variation present in published data, these figures seem to be at a middle level. I was able to obtain a good fit using a surprisingly simple equation for velocity v_4 [m/s] from from a barrel of length b [mm]:

$$v_4 = -71890/b + 1063.8$$

7.62 mm x 54 R Bullet L

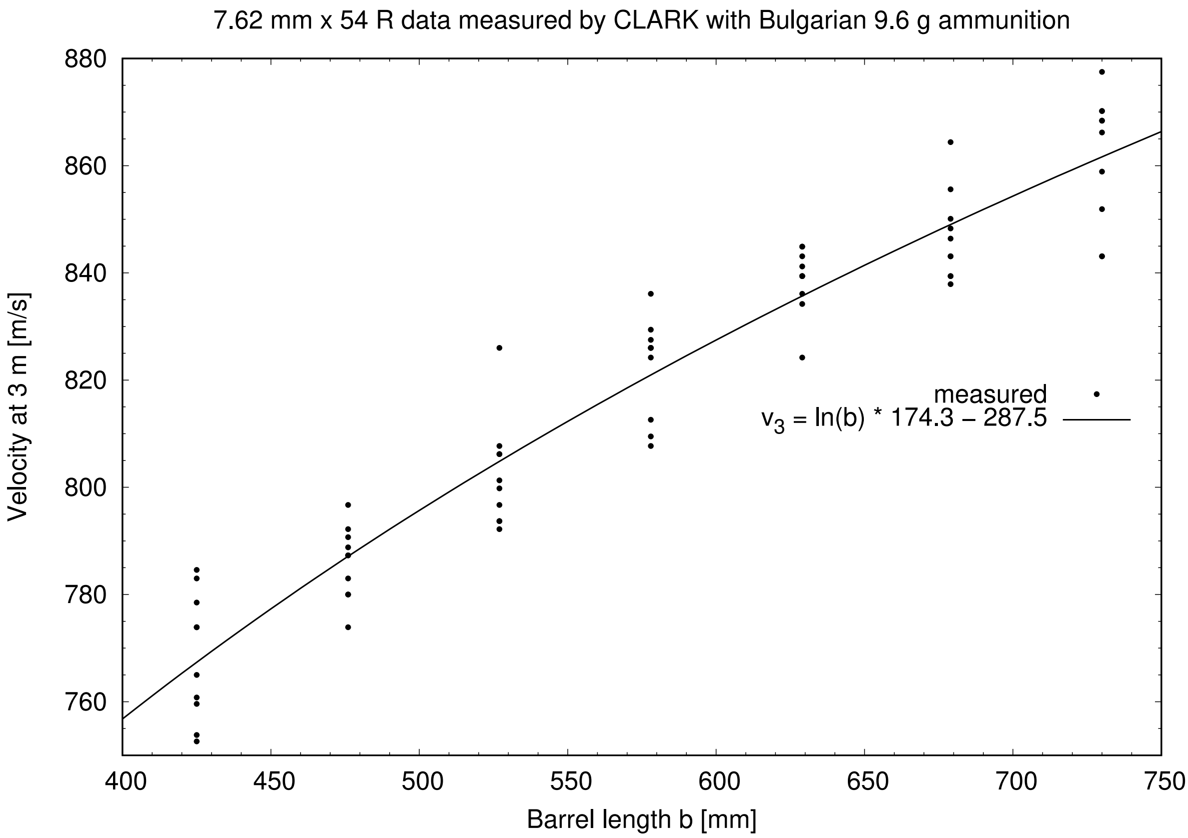

Fig. 12: Tool for estimating 7.62 x 54 R velocity v_3 from barrel length. Based on seventy shots by Clark [5] from a Mosin barrel cut down from 730 to 425 mm. Ammunition was Bulgarian surplus with a 9.6 g bullet.

We owe a lot to Brandon Louis Clark of South Florida State University for having undertaken extremely useful measurements [5] of the change in velocity caused by barrel length. He cut down the same barrel, avoiding bias due to the velocitiy variations inherent in different barrels. Plus he used enough shots (10) to obtain meaningful averages.

Due to the recent presentation of the Russian Tschukawin (Chukavin) sniper rifle with a barrel of only 410 mm, interest in velocity versus barrel length for the Russian 7.62 mm rifle cartridge (called 7.62 x 54 R in the West) has been renewed. Clark's data is very helpful in this respect.

Figure 12 shows a curve fitted to his 70 shots, which is not the straight line Clark uses. The velocities were measured "approximately 10 feet" [5, p. 8 of the PDF version], so the formula found here yields v_3 [m/s] from a barrel of length b [mm]:

$$v_3 = \ln b \times 174.3 - 287.5$$

7.62 NATO Bullet

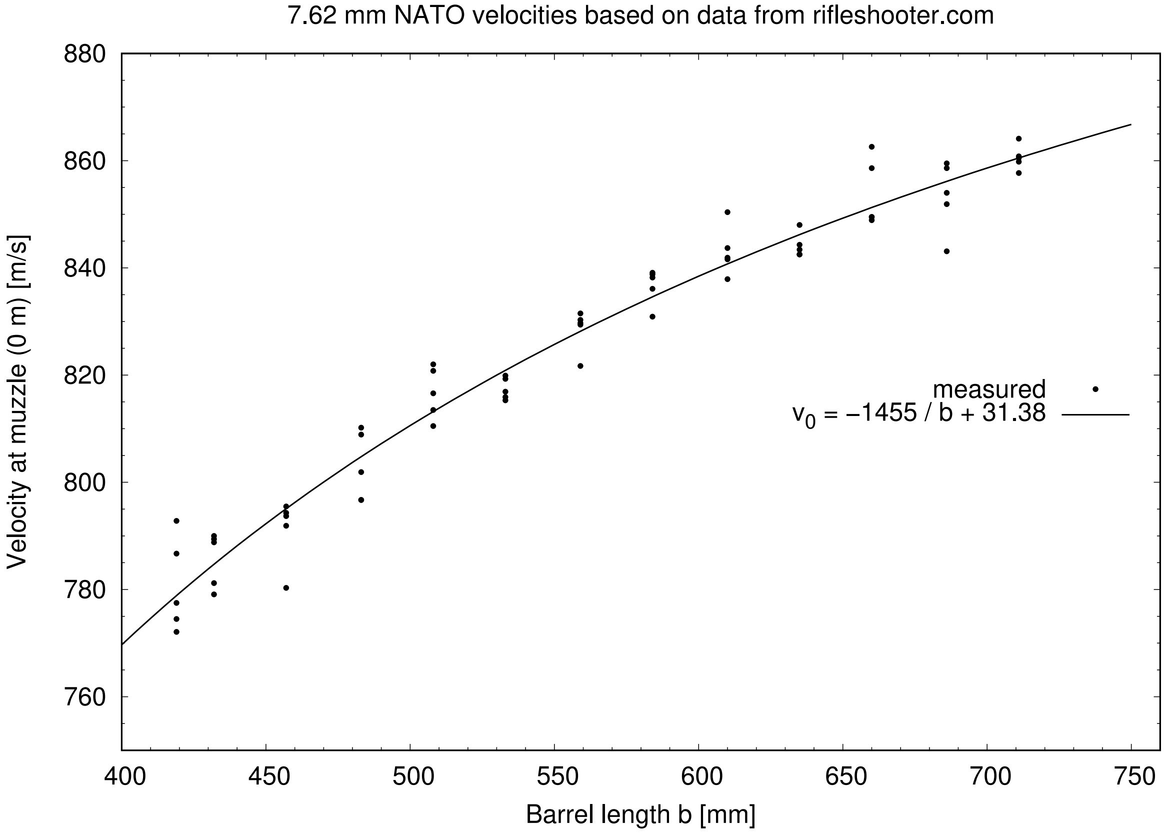

Fig. 13: Tool for estimating 7.62 NATO muzzle velocity v_0 from barrel length. Based on data from `rifleshooter.com` [1], obtained by cutting down a barrel from 711 to 419 mm. Again, a very simple curve fit equation could be found.

On the Rifleshooter Web Site [1] an excellent experiment with .308 Winchester cartridges was published. Like Clark, the author cut down a barrel. The only small drawback was the use of only five shots per barrel length. A definite advantage was the use of a Magnetospeed, which directly reported muzzle velocities, because it is a muzzle mounted chronograph.

Among the tested cartridges, a Winchester load with a 147 gr (9.5 g) full metal jacket (FMJ) bullet was expected to show most similarity to a typical 7.62 NATO. But the velocities were definitely too high, particularly at 660 mm barrel length. An Israeli cartridge with a 150 gr (9.7 g) FMJ bullet showed very good agreement with several NATO cartridge measurements that I have seen or done myself. This is therefore used in figure 13. Muzzle velocity v_0 [m/s] from a barrel of length b [mm] can be estimated by equation:

$$v_0 = -1455/b + 31.38$$

References

[1] *308 Winchester / 7.62x51mm NATO: Barrel Length versus Velocity.* URL: rifleshooter.com/2014/12 (visited 2015-02-01)

[2] A2-222/0-0-4741 – *Das Gewehr G36.* Gültig ab 25. April 2005

[3] ARVIDSSON, Per G: *Weapons & Sensors.* Las Vegas: NDIA Small Arms Conference, 18-20 May 2009; p. 19

[4] ARVIDSSON, Per G: *Is there a problem with the lethality of the 5.56 NATO caliber?* Dallas, TX: NDIA Joint Armaments Conference, 17-20 May 2010

[5] CLARK, Brandon Louis: *Effect of Barrel Length on the Muzzle Velocity and Report from a Mosin-Nagant 7.62x54R rifle.* Tampa: University of South Florida 2011

[6] *Department of Defense Dictionary of Military and Associated Terms.* Washington DC: Joint Chiefs of Staff 2001; as amended through 30 May 2008; AD-A485800

[7] *dto.* 2010; as amended through 15 February 2016; AD-1004650

[8] DWORJANINOW, W[ladislaw] N[ikolajewitsch]: *Bojewye patrony strelkovogo orushija.* 4 volumes, Klimowsk: Isdatelswo "D'Solo" 2015

[9] FM 3-22.9 – *Rifle Marksmanship M16-/M4-Series Weapons.* Washington: Department of the Army, 12 August 2008; including Change 1, 10 February 2011

[10] GAMMON, W J: *Shot Group Statistics for Small Arms Applications.* Quantico, VA: Marine Corps System Command 2017; AD-1034885

[11] GRUBBS, Frank E[phraim]: *Statistical Measures of Accuracy for Riflemen and Missile Engineers.* 2nd edition, Havre de Grace, MD: Frank Grubbs 1991; p.4

[12] ITOP 4-2-829 – *FR/GE/UK/US Vertical Target Accuracy and Dispersion.* Aberdeen, MD: U.S. Army, 23 October 1992,Appendix A; AD-A257489

[13] JOHNSON, Richard F: *Statistical Measures of Marksmanship.* Natick, VA: Military Performance Division 2001; AD-A387108; p. 4-7

[14] LITZ, Bryan: *Ballistic Performance of Rifle Bullets.* 2nd edition, Cedar Springs, MI: Applied Ballistics, 2015

[15] MIL-C-63989A – *Cartridge, 5.56 mm, Ball, M855.* 05 October 1984

[16] MIL-C-63989C – *Cartridge, 5.56 mm, Ball, M855.* 15 February 1999

[17] *dto.* Amendment 3, 18 October 1999

[18] MIL-DTL-63997C – *Rifle, 5.56 Millimeter: M16A2.* with Amendment 4, 17 June 2015

[19] MIL-DTL-70599B – *Carbine, 5.56 Millimeter – M4.* 05 August 2009

[20] PEELEN, Jochem: *Ballistik kleiner Kaliber: der Luftwiderstand.* 2. erweiterte Auflage, Nordersted: Books on Demand 2017

[21] PEELEN, Jochem: *GPMG Rate-of-fire Considerations.* Shrivenham: Small Arms Symposium, 21-22 Aug 2013

[22] Personal communication.

[23] *Precision Rifles [G28, MSG90A2].* Promotional flyer, Oberndorf: Heckler & Koch 2014

[24] RANES, Bethany etal.: *Effects of Rifle Handling, Target Acquisition and Trigger Control on Simulated Shooting Performance.* Fort Rucker AL: U.S. Army Aeromedical Research Laboratory 2014; AD-A601359

[25] SEMIKOLENOW, N P *et.al.*: *Osnowy strelby iz orushija strelkowych podrasdelenij.* Moskwa 1958

[26] *dto.* Translation by U.S. Army Foreign Science and Technology Center 1971; AD-723572

[27] STROHM, Luke S: *An Introduction to the Sources of Delivery Error for Direct-Fire Ballistic Projectiles.* Aberdeen MD: Army Research Laboratory 2013; AD-A588846

[28] STANAG 2310 – *Small Arms Ammunition (7.62 mm).* Bruxelles: NATO [ratified 1957]

[29] STANAG 4172 – *5.56 mm Ammunition (Linked or Otherwise).* Edition 2. Bruxelles: NATO 1993

[30] TC 3-22.9 – *Rifle and Carbine.* Washington: Department of the Army, 13 May 2016

[31] TSCHUMAK, R N: *Russkij 7,62-mm Wintowotschnyj Patron.* Sankt-Peterburg: Atlant 2007